A MIMIC Approach to the Estimation of the Supply and Demand for Construction Materials in the U.S.

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Abstract:

A Multiple-Indicators, Multiple-Cause (MIMIC) approach is used to model the market for traditional construction aggregates. The long-run dynamics of the construction industry is related to the demand for aggregates through a partial adjustment mechanism. The dynamic model for construction serves as the multiple causes specification for the estimation of a simultaneous equations model of the demand and supply of aggregates. A two-step estimation procedure similar to that proposed by Hsiao (1989) for non-linear errors-in-variables models is used. The asymptotic variance-covariance estimates are compared to those from a bootstrapping experiment.

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Introduction

This paper presents the modelling and estimation of the market for two
traditional construction materials, sand and gravel. Sand and gravel are nonfuel
minerals used in construction (buildings, roads, etc.), commonly known as
aggregates. Although aggregates are non-renewable resources, their markets
have received very little attention in past research, most likely due to their
widespread geological (but not economic) abundance.

The aggregates markets are characterised by important spatial delineation
and high transport costs, making the definition of a market crucial to obtaining
meaningful elasticity estimates. The demand side of the market is derived from a
constrained cost minimisation depending on total output and input prices.
Construction activity is the total output. Construction is a flow variable related to
the stock of actual and desired infrastructure at any given time. For modelling
purposes, the demand for construction can be viewed similarly to the household's
demand for natural gas (Balestra and Nerlove (1966)). They argued that at any
given point in time, natural gas demand was highly dependent on the household's
stock of existing appliances. This stock-flow relationship implies that the demand
for the flow variable, construction, has to be related to the underlying dynamic
adjustment of the stock variable, infrastructure. We incorporate the dynamic nature
of construction activity in the estimation of the input demand for aggregates. Actual
infrastructure in place relates to the unobservable desired level of infrastructure
through a partial adjustment mechanism. The desired level of infrastructure stock
in market $i$ at time $t$, can be related the level of population and the per capita income in market $i$ at time $t$.

On the supply side of the market, and for any given period, quantity supplied to the market is different from quantity extracted (produced). Aggregate can be stockpiled until time for sale, making the price of inventory holdings low in this industry. The determinants of quantity produced or extracted during a given year can be related to geological conditions, cost of extraction, and the perceived present value of future income (see for example Conrad and Clark (1987)). In contrast to quantities extracted, the quantity sold in a given year depends on relative (competing) output prices, inventory levels, and market conditions. The full economic model is presented in Section II.

The statistical (supply-demand) model of the aggregates market is defined for a cross-section of markets, the Bureau of Mines' sand and gravel districts in the US. One of the exogenous variables of the demand equation, construction, depends on the underlying dynamics of the stock of infrastructure. Due to data limitations, namely that a pooled time-series and cross sectional data set is not available for the system, we propose the estimation in two steps. First, under certain assumptions discussed in Section II, a construction demand model is estimated. The parameter estimates obtained from this first step are then used to calculate a "predicted" construction activity level for each district. In this model, quantity demanded of sand and gravel in district $i$ at time $t$ is an indicator of an "unobservable" variable, construction activity in district $i$ at time $t$, $C_i$. Through its dynamic adjustment to the stock of infrastructure, $C_i$ is a function of some observable exogenous determinants (or causes), namely population and per capita
income. This approach is known in the literature as the multiple-indicator, multiple-cause (MIMIC) specification. As we develop the model, in Section II, and discuss the econometric implications, in Section III, we will come back to the MIMIC specification (See Goldberger (1974) and Aigner et. al (1984)). The consistency of the two step procedure and the consequences on the asymptotic covariance of the two stage least squares estimator are also addressed in Section III.

Similar ideas have been introduced by Iwata (1992) and Hsiao (1989). Iwata explores the use of estimated latent variables in limited dependent variable models. Hsiao examines consistent estimation for non-linear single equation models with unobservable variables. The organisation of the paper is as follows: In Section II we present the full economic model for this problem. In Section III we present the consistent estimation procedure and explore the consequences on the asymptotic covariance of the two stage least squares estimator. Estimation results are presented in Section IV with a further evaluation of the efficiency of the estimation procedure through a bootstrapping experiment. Conclusions are presented in Section V.

II. The Economic Model

II.1. The Demand

The demand for sand and gravel is an input demand derived from a cost minimisation problem. Let the production function for construction, broadly interpreted, be:

\[ C_{it} = f(S_{it}, O_{it}) \]  \hspace{1cm} (1)

where \( C_{it} \) is total construction activity in region \( i \) at time \( t \); i.e., buildings, highway
construction, etc. $S_n$ is the total quantity of aggregate, and $O_n$ is the total quantity of other inputs used in construction. By minimising cost subject to (1), we obtain the input (or factor) demand for sand and gravel,

$$S_{ict} = d(C_{ict}, P_{ict}, w_{ict})$$

where $P_n$ is the fob price of sand and gravel, $w_n$ is the price of other inputs, and $S_{ict}$ is the input demand for aggregate.

If (1) is a Cobb-Douglas production function, then (2) takes the form

$$S_{ict} = (C_{ict})^{1/a} \cdot A \cdot \left( \frac{w_{ict}^{b_1}}{P_{ict}^{b_2}} \right)^{b_2}$$

where $S_{ict}$ is the input demand function for cross-sectional unit $i$ at time $t$, and $a = b_1 + b_2$ and $A$ is a constant.

As discussed in the introduction, construction activity is a flow variable with an underlying relationship to the stock of infrastructure. Let the actual level of infrastructure stock be $H_n$, then

$$H_{ic} = (1-\delta) H_{i,c-1} + C_{ict} \quad 0 < \delta < 1$$

where $C_n$ is total construction activity and $\delta$ is the depreciation of infrastructure stock. Let the desired level of infrastructure stock in $i$ at time $t$, $H_{it}^*$, depend linearly on the population and per capita income in region $i$ at time $t$:

$$H_{it}^* = (1 \text{ pop}_{ic} \: pcinc_{ic})' \beta$$

where $\beta$ is a 3x1 parameter vector.

Suppose that the adjustment of the actual infrastructure level, $H_{it}$, follows a proportional adjustment to $H_{it}^*$:
By (4), (5), and (6), we can derive the following reduced form (detailed derivation in Appendix 1).

\[ H_{it} - H_{i,t-1} = \gamma (H_{it}^* - H_{i,t-1}^*) + \varepsilon_{it} \quad (6) \]

The difference equation in (7) is a model for construction activity. We will assume that preferences for type of construction are homogenous across districts and states. This assumption implies that, other things the same, taste for construction is constant across districts and therefore the parameters in (7) are constant across cross sectional units (i).

\[ C_{i,t} = \pi_0 + \pi_2 \text{pop}_{i,t} + \pi_3 \text{pcinc}_{i,t} + \pi_4 C_{i,t-1} + \pi_4 C_{i,t-2} + \ldots + \varepsilon_{i,t} \quad (7) \]

\[ \varepsilon_{i,t} \sim (0, \sigma^2) \]

II.2. The Supply

The supply of aggregates to a particular market will crucially depend on transportation costs. However at mine mouth, the determinants would be the fob price, the costs of production, and availability of inventories. Costs of production are difficult to obtain, since producers are reluctant to give out what is considered confidential information and supply conditions vary geographically. Sand and gravel pits and crushed stone quarries often coexist in the same area. The equipment needed for some of the processes is similar (washing, screening, grading, and transportation equipment). Moreover, many companies own both sand and gravel pits and stone quarries. This degree of rivalry in the use of the same inputs of production between sand and gravel and crushed stone can be exploited for modelling purposes.
Output prices can, under certain assumptions, be expressed as functions of input prices. Jorgenson and Fraumeni (1981) call this function as the "sectoral price function." Therefore, output price is a reflection of input use and substitution. This has been utilised several times in natural resources by modelling quantity supplied of a given product as a function of the ratio of output prices, the own price to the price of the output rival in the use of factors of production. By this argument, we expect changes in the price of crushed stone to shift the supply of sand and gravel. We must, however, recognise that stone may be substituted for gravel in some uses. The substitution mainly occurs in asphaltic concrete, where angular or flattish fragments do not compromise stability. However, concrete aggregate, used for residential and nonresidential construction, requires naturally round particles to insure strength, making crushed stone undesirable for these uses (Evans, 1978).

The model of the supply side of the sand and gravel market is in general defined as

\[ \bar{s}_{i,t} = s \left( \frac{p_{i,t}}{q_{i,t-1}} \right) \]  

where \( \bar{s}_{i,t} \) is the supply of sand and gravel and \( q_{i,t-1} \) is the lagged price of crushed stone.

11.3 The Market Model

If we take equation (2), (7), and (8) and assume log-linear functional form, then the full economic model for the sand and gravel market is given by:
\[
\begin{align*}
\ln(s_{it\delta}) &= \alpha_{i1} + \alpha_{i2} \ln(x'_{it\delta} \pi + \varepsilon_{i\delta}) + \alpha_{i3} \ln(w_{it}) + \phi_{i1} \ln(p_{it}) + \mu_{i\delta}, \\
\ln(s_{it\delta}) &= \alpha_{i1} + \alpha_{i4} \ln(q_{1, it-i}) + \phi_{i2} \ln(p_{it}) + \mu_{i\delta}, \\
\ln(s_{it\delta}) &= \ln(s_{i\delta}),
\end{align*}
\]

where \( x'_{it\delta} \pi = C_{it} = \pi_0 - \pi_1 p_{it} + \pi_2 p_{inc_{it}} + \ldots \), and it is assumed \( \mu_{i\delta} \sim (0, \sigma^2) \).

This market model is defined at mine mouth, that is \( p_{it} \) is the fob price of sand and gravel before transport cost, and \( s_{it} \) is the quantity sold in district \( i \) at time \( t \). This price specification presumes that delivered price is the same proportion of fob price across observations, a strict but unavoidable assumption due to level of data on transport distances and costs.

Possibilities for consistent estimation of (9) depend on whether the system is identified. Let's assume for the sake of argument that \( C_{it} \) is observable. It can be easily verified by the traditional rank and order conditions, that the demand equation is just-identified and the supply equation is overidentified. Given that \( C_{it} \) is not directly measured at the district level, then the traditional rank and order condition are not useful in this case. In general, in the presence of unobservable variables, overidentifying restrictions can be traded-off against measurement error variances in many cases. Gerarci (1977) developed a rank condition for identification of systems of equations with measurement error under the assumption of multinormality. The fundamental rank condition (Gerarci, 1977) depends on the location of exogenous variables measured with error and on the overidentifying restrictions appearing elsewhere in the system. It can be shown that the conventional rank condition is necessary but not sufficient when variables
are measured with error. Hsiao (1989) argues that for errors-in-variables models, the existence of consistent estimators implies identifiability, and the existence of such consistent estimators is a sufficient condition for identification.

III. A Consistent Estimation Procedure

Without a panel of data, the model in (9) cannot not be estimated. However, by a combination of economic assumptions and available information (i.e., time series) consistent estimation is possible in a two step procedure:

1. Consistent estimation of a construction activity model, (7), using available time series information at a more aggregated level. This first step involves the estimation of the multiple-cause part of the model. A consistent estimator for (7) is OLS if $\varepsilon_t$ is not serially correlated.

2. Use the estimated $\pi$, $\hat{\pi}$ from (7), to predict construction activity for each district. This requires the homogeneity assumption made in Section II. Then, use 2SLS on (10) for the cross section of sand and gravel districts. This step yields estimation of the multiple-indicator part of the model.

\[
\begin{align*}
\ln(s_{ict}) &= \alpha_{21} + \alpha_{24}\ln(q_{i,t-1}) + \phi_{21}\ln(p_{ict}) + \mu_{ict} \\
\ln(s_{ics}) &= \alpha_{11} + \alpha_{12}\ln(c_{ict}) + \alpha_{13}\ln(w_{ict}) + \phi_{11}\ln(p_{ict}) + \mu_{ics}
\end{align*}
\]  

As long as $\pi$ is consistently estimated, replacing it by $\hat{\pi}$ for the second step does not affect the consistency of the 2SLS. Consistency is ensured by the independence of $\hat{\pi}$ and $\mu_{it}$. Further, as all the parameters in (9) can be
consistently estimated, it follows, from Hsiao's argument, that the system is identified.

To analyse the consequences on the asymptotic covariance of the 2SLS estimator of an "estimated" exogenous variable, we need to concentrate on $X$, the matrix of all the exogenous variables in system (9), and $X_r$ the matrix of included exogenous variables in equation $i$ ($i=1,2$). The existence of the asymptotic covariance matrix of the 2SLS estimator depends mainly on the convergence of $(X'X)/T$, $(X_r'X_r)/T$, and $(X'X_r)/T$ to finite matrices say, $Q_x$, $Q_{x_r}$ and $Q_r$ respectively. While the formal discussion is presented in Appendix 2, the conclusion is that provided $X$ (just defined) and $\mu$ (in (9)) are independent, the asymptotic covariance of the 2SLS can be estimated as:

$$\text{Cov}(\hat{\delta}_i) = \sigma_i^2 [EZ_i'EZ_i]^{-1}$$

$$\hat{\sigma}_i^2 = \frac{(Y_i-EZ_i\hat{\delta}_i)'(Y_i-EZ_i\hat{\delta}_i)}{T}$$

where $EZ_i = [\hat{Y}_i, \hat{X}_i]$ , $EZ_2 = [Y_i, \hat{X}_i]$ , $\hat{X}_i$ is the matrix of exogenous variables with construction replaced by its predicted value, $Y_i$ is the matrix of endogenous variables, price and quantity, and $\hat{\delta}_i$ the 2SLS estimator, $i=1,2$. 

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IV. The Empirical Results

IV.1. The Data

The time series data for construction were obtained from Construction Report (U.S. Department of Commerce, varying years) at the U.S. level, and consisted of annual data on the values of residential construction, nonresidential construction, and additions and alterations by permit issuing places. A price deflator for the construction industry and annual data on population and per capita income for the U.S. were from Citibase (Citicorp Economic Database).

The cross-sectional data were obtained from several sources as well. Average construction workers wages (i.e., price of labour, $w_i$, in (9)), population, and per capita income at the county level were obtained from the Regional Economic Information System for the period 1982-1989 (REIS). Sand, gravel, and crushed stone data at district levels were obtained from the Bureau of Mines, Minerals Yearbook, Vol.II, Area Reports for 1987 and 1988. Sand and gravel data have been published at the district level for 1988 and crushed stone data have been published at this level for 1987. The published information contained total quantity (in thousands of short tons) and total value (in thousands of dollars) sold by district for construction purposes. Sand and gravel are reported jointly. The Bureau designated an average of 4 districts in each one of 40 states of the U.S. by grouping adjacent counties. Using Fips county codes, we mapped the county level data (over 3000 observations) into districts. City-County Data Book, 1988, which is published every 5 years, provides total value of construction (residential, non-residential, additions, and alterations) by permit issuing places at county level for 1986. The REIS and County and City Data Book data are coded by Fips codes.
The complete district level data set contained a total of 165 districts in 40 states. Whenever a particular district had less than 3 operations the Bureau did not publish the information to protect particular operations. Therefore, many districts had missing values.

VI.2. The Estimation

Estimation of the reduced form for Construction Activity (eq. (7)) was performed with annual time series data. Individual series for population, per capita income, and construction were tested for unit roots using Augmented Dickey-Fuller test statistics (Results table in Appendix 3). Per capita income was found to be non-stationary, and therefore changes in per capita income were used for the estimation. The criteria to truncate the number of lags were parsimony and presence of serial correlation in the residuals. The estimated model, with probability values in parenthesis, is:

\[ \hat{C}_i = -78755 + 0.6595\text{pop}_i + 20.592\Delta\text{pcinc}_i + 0.36756C_{i,1} \]  \hspace{1cm} (12)

\( R^2 = 0.8697, T=29 \)

Durbin H Statistic = 0.49537 (asymptotic normal)

Box-Pierce-Ljung Q(12) = 10.81 \( (\chi^2_{(a=5\%)} = 21.026) \)

Box-Pierce-Ljung Q(24) = 21.33 \( (\chi^2_{(a=5\%)} = 36.415) \)

From (12) it follows that \( \pi' = (-78755 \ 0.6595 \ 20.592 \ 0.36756) \). To find an "estimated" value for construction in district i, cross sectional data for population, per capita income, and the available lagged year construction value at the county level were used. District i estimated value of construction is:

\[ \hat{C}_i = -78755 + 0.6595\text{pop}_i + 20.592\Delta\text{pcinc}_i + 0.36756C_{i,1} \]
The "estimated" $\hat{c}$ was incorporated for the second step (i.e., estimate of the demand and supply for sand and gravel through eq. (10) by 2SLS).

Although system estimators are asymptotically more efficient, any specification error in the structure of the model will be propagated throughout the system by 3SLS or FIML (see Greene, 1990 pages 636-638, for a detailed discussion). In our particular case, the short-run supply function for a non-renewable resource would depend upon mining costs and the ratio of inventories to production. The limitations of data may lead to misspecification of the supply equation because of these omitted variables. A system estimation is most likely to propagate the misspecification to the estimation of the demand. We decided to use 2SLS, although the interested reader will find the 3SLS estimates reported in Appendix 4.

The first two columns of Table 1 show the 2SLS estimates for eq. (10), and the estimated standard errors calculated from (11). The third and fourth columns of Table 1 show the results of a bootstrapping experiment. Bootstrapping is a technique for estimating standard errors using a Monte Carlo simulation based on a nonparametric estimate of the underlying error distribution. The basic distinction from the classical approach is that the bootstrap uses simulation instead of asymptotics (Freedman and Peters, 1984a). Freedman and Peters (1984a and 1984b) describe in detail the procedure to obtain coefficient estimates and estimated standard errors for OLS, GLS, 2SLS, and 3SLS.
Table 1

Supply and Demand for Sand and Gravel: 2SLS Estimates and Bootstrap Experiment

<table>
<thead>
<tr>
<th></th>
<th>2SLS Estimate</th>
<th>2SLS SE</th>
<th>Bootstrapped Mean</th>
<th>Bootstrapped SD</th>
<th>bias</th>
<th>t**</th>
<th>F***</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DEMAND:</strong> Intercept</td>
<td>-2.1013</td>
<td>1.888</td>
<td>-2.2822</td>
<td>2.9226</td>
<td>-0.1809</td>
<td>-0.7662</td>
<td>2.396265</td>
</tr>
<tr>
<td>In((\tilde{c}_i))</td>
<td>0.2183</td>
<td>0.072</td>
<td>0.2149</td>
<td>0.1302</td>
<td>-0.0034</td>
<td>-0.3564</td>
<td>3.260031</td>
</tr>
<tr>
<td>In((\tilde{p}_i))</td>
<td>-0.7989</td>
<td>1.6808</td>
<td>-0.6298</td>
<td>4.1315</td>
<td>0.1691</td>
<td>0.6530</td>
<td>6.040575</td>
</tr>
<tr>
<td>In((\tilde{w}_i))</td>
<td>2.6169</td>
<td>0.8128</td>
<td>2.6233</td>
<td>1.1775</td>
<td>0.0064</td>
<td>0.0643</td>
<td>2.096938</td>
</tr>
</tbody>
</table>

| **SUPPLY:** Intercept| -2.2717       | 5.9333  | -1.3897           | 6.6363          | 0.882 | 1.2993 | 1.250893 |
| ln(\(\tilde{p}_i\))| 10.808        | 6.1528  | 9.7952            | 6.8273          | -1.0128 | -1.4412 | 1.231159 |
| ln(\(\tilde{q}_{i+1}\))| -1.9490       | 1.4667  | -1.7397           | 1.531           | 0.2093 | 1.2646 | 1.090598 |

N = 450 replications

** t-stat for H₀: \(\bar{\delta} - \hat{\delta} = 0\) against H₁: \(\bar{\delta} - \hat{\delta} \neq 0\)

** F-stat for H₀: \(s^2_\delta = s^2_{\hat{\delta}}\) against H₁: \(s^2_\delta > s^2_{\hat{\delta}}\)

Results indicate that supply is highly elastic and demand is inelastic, as expected. The share of aggregates in total construction cost is small, making demand relatively inelastic. Due to the possibility of large inventory holdings, supply was expected to be relatively elastic. However, as we were unable to hold capital and land fixed across districts, due to data limitations, the estimated supply could be viewed as a long-run supply.
The fifth and sixth column of Table 1 report the estimated bias and the t-statistics for the estimated bias. The bias is estimated as the difference between the bootstrap estimate (column 3) and the 2SLS estimated value (column 1). The t-statistics for the hypothesis of zero bias are calculated as

$$t^* = \frac{\hat{\delta} - \hat{\delta}_{0}}{s_{\hat{\delta}} + s_{\hat{\delta}}^2 / \sqrt{450 + 97}}$$

and reported in column 6 of Table 1. We fail to reject the null hypothesis for all estimated bias values, and conclude that the 2SLS estimator in this application is negligible biased. The F-statistic to test the hypothesis that the bootstrapped standard errors are equal to the asymptotic estimate is reported in column 7. The F values were larger than the critical value in all cases except for $\ln(q_{t-1})$ in the supply equation (marginal rejection for $\ln(p_t)$). Thus, the evidence suggest that estimated asymptotic standard errors are downward biased for the demand equation, but not for the slope parameters of the supply equation.

Finally, the variance-covariance matrix of the bootstrapped estimates for the supply and demand equations were calculated and denoted $\Sigma_{s2}$ and $\Sigma_{d2}$, respectively. The variance-covariance of the 2SLS estimates for the supply and demand were denoted $\Sigma_{s1}$ and $\Sigma_{d1}$, respectively. The following null hypotheses were tested: $H_0: \Sigma_{s2} = \Sigma_{s2} = 0$ and rejected in both cases using Nagao (1973) alternative invariant version of the likelihood criterion. Although individual F-tests indicated that only the demand equation asymptotic standard errors were biased, the joint tests for the variance-covariance indicates the existence of bias for both equations.
V. Conclusions

This paper models the supply and demand for the aggregates markets. Data limitations prevented the estimation of a full time series and cross-sectional simultaneous equations model. A consistent estimation procedure is proposed and used. Asymptotic results are compared to those of a bootstrapping experiment. Estimated price elasticities indicate the demand for aggregate is relatively inelastic and supply relatively elastic. The bootstrap experiment shows that the bias of the 2SLS is not significant, and the estimates obtained with asymptotic formulas for estimating the standard errors are biased downwards for the demand equation.


Appendix 1

Derivation of reduced form for construction activity, eq. (7)

By successive substitution in (4) we obtain

\[ H_n = (1-\delta)[(1-\delta)H_{i,t-2} + C_{i,t-1} + C_{i,t}] \]

\[ H_n = \sum_{j=0}^{\infty} (1-\delta)^j C_{i,t-j} \quad (1A) \]

By adding \( \gamma H_{i,t-1} \) to both sides of (6) we obtain

\[ H_n - (1-\gamma)H_{i,t-1} = \gamma H_{i,t} + \epsilon_n \quad (2A) \]

Substituting (1A) and (5) into (2A) and rearranging:

\[ C_n = \gamma \beta_o + \gamma \beta_1 \text{pop}_{i,t} + \gamma \beta_2 \text{pcinc}_{i,t} + \]

\[ [(\delta-\gamma)/(1-\delta)] \sum_{j=1}^{\infty} (1-\delta)^j C_{i,t-j} + \epsilon_n \quad (3A) \]

or

\[ C_n = \pi_o + \pi_1 \text{pop}_{i,t} + \pi_2 \text{pcinc}_{i,t} + \pi_3 C_{i,t-1} \]

\[ + \pi_4 C_{i,t-2} + \ldots + \epsilon_n \quad (7) \]
Appendix 2

The Asymptotic Covariance of the 2SLS Estimator

Let \( \Pi_i \) be the matrix of reduced form parameters in equation \( i \), \( X \) the matrix of fixed exogenous and predetermined variables in the system, \( X_i \) the matrix of fixed exogenous and predetermined variables included in equation \( i \), \( Y \) the matrix of endogenous variables in equation \( i \), and \( \hat{\delta}_i \) the 2SLS estimator.

The asymptotic distribution of the 2SLS estimator is given by:

\[
\sqrt{T}(\hat{\delta}_i - \delta_i) \overset{a}{\sim} N(0, \sigma_i D_i^{-1}) \tag{4A}
\]

where, \( D_i = \begin{pmatrix} \Pi_i \Omega_i \Pi_i & \Pi_i \Omega_i \\ \Omega_i \Pi_i' & \Omega_i \end{pmatrix} \)

\[
Q_x = \lim_{T \to \infty} \left( \frac{X'X}{T} \right) , \quad Q_x = \lim_{T \to \infty} \left( \frac{X'X_i}{T} \right) , \quad Q_i = \lim_{T \to \infty} \left( \frac{X'X_i}{T} \right) , \quad \sigma_i = E(\mu_i'\mu_i) \\
\]

and \( \mu_i = \gamma_i - \chi_i \gamma_i - \chi_i \beta_i \)

The system of equations in (9) differs from the traditional case in that one of the exogenous variables in the demand equation is not fixed but random. Under the assumptions that \( \epsilon \) (error term of the multiple-cause part of the model) and \( \mu_i \) (error term of the simultaneous equation) are independent, and construction activity and \( \mu_i \) are independent, we only require,

\[
Q_x = \text{plim}\left( \frac{X'X}{T} \right) \quad Q_{x} = \text{plim}\left( \frac{X'X_i}{T} \right) \quad Q_i = \text{plim}\left( \frac{X'X_i}{T} \right) \quad \text{all finite.}
\]
It follows that the limiting distribution in (4A) still holds as long as \( D_i \) is non-singular, and the estimated covariance of the 2SLS is:

\[
\hat{\text{Cov}}(\hat{\delta}_i) = \hat{\sigma}_i^2 (\hat{E}Z_i' \hat{E}Z_i)^{-1}
\]

where \( \hat{E}Z_i = [\hat{Y}_i \hat{X}_i] \), \( \hat{E}Z_i = [Y_i X_i] \), \( \hat{X}_i \) is the matrix of exogenous variables in the demand equation with construction replaced by its predicted value, \( Y_i \) is the matrix of endogenous variables, price and quantity, \( \hat{Y}_i \) is the matrix of predictors obtained from the estimated reduced form of \( Y_i \), and \( \hat{\delta}_i \) the 2SLS estimator.

In summary, the requirement is that \( \text{plim}(X'X)/T \), \( \text{plim}(X'X)/T \), and \( \text{plim}(X'_iX_i)/T \) replace \( \text{lim}(X'X)/T \), \( \text{lim}(X'X)/T \), and \( \text{lim}(X'_iX_i)/T \).
Appendix 3

Dickey-Fuller and Augmented Dickey-Fuller Unit Root Test

<table>
<thead>
<tr>
<th>Test</th>
<th>Critical Value (5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction Permits (Augmentation lag=1')</td>
<td></td>
</tr>
<tr>
<td>$t^*$</td>
<td>-2.435</td>
</tr>
<tr>
<td>$\Phi_1$</td>
<td>3.245</td>
</tr>
<tr>
<td>$t^\alpha$</td>
<td>-4.876</td>
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<tr>
<td>$\Phi_2$</td>
<td>8.228</td>
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<td>Population (Augmentation lag=1')</td>
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<tr>
<td>$t^*$</td>
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<tr>
<td>$\Phi_1$</td>
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<tr>
<td>$t^\alpha$</td>
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<td>$\Phi_2$</td>
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<td>Per Capita Income (Dickey-Fuller, lag=0')</td>
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<tr>
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<td>46.271</td>
</tr>
<tr>
<td>$\Phi_3$</td>
<td>3.742</td>
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* Determined by AIC and SIC  
** Critical Values are from Fuller, 1976 pp. 373 and Dickey and Fuller, 1981 pp 1063.
## Appendix 4

### Supply and Demand for Sand and Gravel: 3SLS Estimates

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<td>$s_{\hat{\delta}}$</td>
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<td><strong>DEMAND:</strong></td>
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<td>Intercept</td>
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<td>$\ln(c_i)$</td>
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<td>$\ln(p_i)$</td>
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<tr>
<td>$\ln(w_i)$</td>
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<td>0.8093</td>
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</tbody>
</table>

| **SUPPLY:**          |          |     |
| Intercept            | -2.271   | 5.9491 |
| $\ln(p_i)$           | 10.808   | 6.1692 |
| $\ln(q_{i,t-1})$     | -1.949   | 1.4706 |

**N** 97
Endnotes

1. FOB prices are prices before transport costs.

2. The Cobb-Douglas production function has been extensively used in resource economics. Kemp and Long (1980) offer additional justification for its use in the context of exhaustible resources.

3. We will assume that the available information set at time \( t \) consists of knowledge of past values of all the relevant variables that determine \( C_t \).

4. An alternative specification of (5) is that where the desired infrastructure level is taken to be a random variable. Under this alternative, an error term would be added in (5) not in (6). This specification can be found in Judge, et al (1985). The consequences of choosing either alternative are discussed in Drymes (1971).

5. Coexistence of both activities has been documented by the Committee on Surface Mining and Reclamation (1980).

6. Formally, we require \( \text{plim} (\hat{\mu}, \mu) / T = 0 \)

7. \( F_{0.05(60,-)} = 1.32, F_{0.05(120,-)} = 1.22 \)

8. See T.W. Anderson Ch. 10


A Note on a Bayesian Estimator in an Autocorrelated Error Model. William Griffiths and Dan Dao, No. 3 - April 1979.


Bayesian Econometrics and How to Get Rid of Those Wrong Signs. William E. Griffiths, No. 31 - November 1987.


